Does Surprisal Predict Code Comprehension Difficulty?

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Abstract

Recognition of the similarities between programming and natural languages has led to a boom in the adoption of language modeling techniques to improve tools to assist developers. However, the measure of surprisal, which guides the training and evaluation of many of these methods, has not been validated as a measure of cognitive difficulty in programming language comprehension as it has for natural language. We perform a controlled experiment to evaluate human comprehension on fragments of source code that are *meaning equivalent* but with *different surprisal*. We find that more surprising versions of code take humans longer to finish answering correctly. We also provide practical guidelines to design future studies for code comprehension and surprisal.

Keywords: Code Comprehension; Language Model Surprisal; Transformer Model

Introduction

As software has become nearly ubiquitous, people, even those who do not consider themselves to be developers, interact with code and learn to program. One of the largest costs in software engineering comes from maintaining existing code (Banker, Datar, Kemerer, & Zweig, 1993); understanding code that others have written (or returning to code a person has written themselves) takes up a large portion of a programmer's time (Tiarks, 2011).

Though code comprehension research has a long history (Siegmund, 2016), there have been more recent calls by psycholinguists to explicitly understand the cognitive processes that drive programming (Fedorenko, Ivanova, Dhamala, & Bers, 2019). Code is often treated as just another kind of language; people refer to teaching coding in terms of "literacy" and describe its structure using terms from linguistics such as grammar, syntax, and semantics. This acknowledgement of the human communicative element of programming is not new: Knuth's Literate Programming emphasizes writing code not for machines, but for the benefit of other developers (Knuth, 1984). While the degree to which natural and programming language share cognitive processes is unknown, recent work has shown some regions of the brain used in natural language processing are also used in programming (Siegmund et al., 2014). In contrast, eye tracking studies have shown people read code differently than natural language; their eyes jump around non-linearly, and this effect increases with experience (Busjahn et al., 2015).

Appreciation of the similarities and differences between natural and programming languages have led to the adoption of computational language models for code. Programming language is more repetitive and has lower surprisal than natural language, which makes these language models *even more effective in a source code context* (Hindle, Barr, Su, Gabel, & Devanbu, 2012), allowing for their adoption to many applications. Automatic completion of code, finding defects and errors, or generating documentation from code are all examples of the kinds tools making a real impact - see (Allamanis, Barr, Devanbu, & Sutton, 2018) for an extensive survey. These tools often leverage language model surprisal and cross entropy as measures to train these tools. Moreover, the repetitive nature of code persists between many natural and programming languages, and some of this is contingent on human choices, above and beyond that which would be expected by inherent differences in language structure (Casalnuovo, Sagae, & Devanbu, 2018). This is suggestive of surprisal having an impact on human comprehension of code.

Though these tools and methods have obtained wide acceptance, the underlying measures of surprisal and entropy used by language models have thus far seen very little validation as relating to what makes code "better" for humans. This lack of testing of assumptions and validating tools is a longstanding problem (Siegmund, 2016). For example, often times when assessing code readability, experiments have relied on developer's *perception* of code, but this is separate from how easy the code is to actually understand (Scalabrino et al., 2017). In natural language, the relationship between language model surprisal and cognitive load is fairly established; higher surprisal leads to higher load (Levy, 2008; Smith & Levy, 2013), but this is not true for code.

In code, this is complicated by developers needing to simultaneously write code for two 'audiences'— one to the machine, which obtains precise semantics through execution, and the other audience is to other humans. These effects intermingle in complex ways, with some elements of code, such as variable names, whitespace formatting, or parenthesis use typically possessing no meaning for the machine channel - they communicate only to other humans (Casalnuovo, Barr, Dash, Devanbu, & Morgan, 2020).

Thus, for code, two expressions can have *identical computational meaning*, but be written in different ways. For example, consider the statement $a = b/c * d;$. This could be equivalently written as $a = (b/c) * d;$, which clarifies the order of operations to the developer, but has no effect on the meaning to the machine. Alternatively, consider how the common code idiom for incrementing is usually written as $i = i + 1;$ and not $i = 1 + i;$. Developers may choose to write one over the other due to either readability concerns or possibly the pressures of existing style or convention.

This feature enables opportunities to explore surprisal in code via controlled experiments. By looking at source code

snippets with different surprisal, but equivalent meaning, we can test hypotheses about whether surprisal can measure the readability and understandability of code. Most related to this study is the recent work by Casalnuovo et al. (Casalnuovo, Lee, Wang, Devanbu, & Morgan, 2019), which looks at the relationship of *human preference* and surprisal. They found humans tended to prefer code with lower surprisal in a forced choice experiment between two lines with different surprisal but identical computational meaning. We use similar methodology and transformations in our study.

In contrast, we wish to look at how surprisal influences *human comprehension* of source code. One way to measure if someone understands code is if they can execute it: given some input can they correctly describe the output? If we have two snippets of source code with equivalent meaning but different surprisal, we can ask humans to compute the outcome of each of them. Thus, we ask two primary research questions to see how easily they understood each variant: how *accurately* do humans compute the answer, and how *quickly* do they *correctly compute* the answer?

Methodology

Materials

Data To develop meaning-equivalent source code fragments, we first train a language model to predict the surprisal of code. Our training and test data comes from the 1000 most starred Java open source projects on Github (`https://github.com/`). From this set, we selected a smaller sample of the 30 Java projects with the most opportunities to perform meaning preserving transformations. We split these into 21 and 9 projects for the training and testing set, chosen randomly, with some preprocessing to remove potentially duplicate or highly similar files by removing those with identical filename and parent directories.

Language Model Training We use a Transformer Model (Vaswani et al., 2017) with Byte-Pair Encoding (BPE) (Sennrich, Haddow, & Birch, 2016) to train the model and obtain surprisal scores for each line. BPE creates subtokens of roughly similar frequency from the tokens, reducing vocabulary size to 9165 subtokens. Code has a larger vocabulary than natural language, which makes training with neural models difficult, and BPE has proven effective at addressing this vocabulary size problem (Karampatsis & Sutton, 2019). This model is implemented in TensorFlow (Abadi et al., 2016), using 2 layers and dimensions of 512 for the attentional and feedforward layers, 8 attention heads, and a dropout rate of 0.1. We train for 10 epochs, with learning rate 0.2, a batch size of 15000, and 200 tokens per sequence.

Meaning Preserving Transformations We use 4 categories of meaning-preserving code transformations. Table 1 shows all our transformation with examples in pseudocode. At the top, we have two types of operations swapping around arithmetic and relational operators. For the arithmetic swaps, we look at $+$ and $*$ operations, which are commutative, and conservatively swap only in cases using numerical variables and literals, avoiding expressions with functions as they may contain side effects that change code meaning. For relational operators, we swap the operands around $==$, $!=$, $>$, $>=$, $<$, and $<=$. If the relation is not symmetric (less and greater than variants), we also invert the operator when swapping to maintain precise meaning. We also add and remove parentheses that are not essential to the meaning of the code, but are often added by developers to be included for readability. These examples appear in the bottom two rows of Table 1.

Table 1: Pseudocode examples of transformations. First column is the general type of operation, second lists the operators involved, and the last two show an example.

Swap Arithmetic	*	a * b	b * a
	+	a + b	b + a
Swap Relational	==, !=	a != b	b != a
	<, <=, >, >=	a <= b	b >= a
Adding Parenthesis		a + b * c	a + (b * c)
Removing Parenthesis		a + (b * c)	a + b * c

Experimental Materials Selection and Validation We selected a total of 64 pairs of original and transformed lines of code (16 from each transformation) to present to study participants, using a combination of initial random sampling and then manual selection, choosing equal numbers of items in which the transformation increases and decreases the surprisal of the line relative to what was written originally. We randomly sampled 64 examples for each transformation from beyond the median in both the positive and negative direction to get examples with high surprisal differences. We filtered these to automatically exclude expressions that were likely to be automatically generated (e.g. hashes), contained rare operations (like bit shifts), were overly easy (comparisons to null or 1), or were over 80 characters long. From these we manually selected 16 samples per transformation, so that no 2 examples would be too similar, and avoided samples that required too much contextual information (such as calls to anything but the most basic functions, like size() on a list).

Next, we artificially created concrete values to initialize all variables used in the expression. These initializations were used for both versions of the expression, and if there were more than 1 variable to initialize, the order was randomized. We used simple initializations for the variables in each expression, to reduce cognitive overhead, but also such that participants would have to evaluate the entire expression. Once these initializations were generated, we ran the code for each of the 64 pairs to verify the correct answer and that they were equivalent. Figure 1 shows an example of an initialization and pair of snippets.

Procedure

Our experiment consists of presenting subjects with 32 randomly chosen samples from our 64 pairs. Each subject is

```
int width = 7;
int x = 3;
// Original Source Code
if(x >= width / 3 * 2) {
// Transformed Source code
if(x >= 2 * width / 3) {
```

Figure 1: Example initialization and expression pair. Participants saw only one of the pair (without the Original/Transformed label) and the multiple choice question: "Does the expression evaluate to true or false?"

randomly shown only 1 variant of the pair, to prevent any learning effect from seeing both versions. For each sample, the subject is first shown the variable initializations for 3 seconds, after which they can advance the page to see the expression that uses them in addition to the initializations.

Then, they are asked to mentally compute the value of the expression after execution. Tasks included computing a numerical value, determining whether a boolean expression was true or false, or determining how many times a `for` loop would execute. For numerical questions, respondents entered the value in a text box, and for boolean values they choose between true/false buttons. For text questions the cursor began in the text box, and true/false questions could be answered with the 't' and 'f' keys, enabling subjects to complete the experiment entirely using their keyboard.

During the experiment, we measured both answer correctness and response time. Correctness is straightforward, though we give credit for similar answers (i.e. "8" counts for a question with a floating point answer of "8.0"). For timing, we used the high precision timing Javascript library Performance[1] to record the timing of every keystroke and click involving the text box or answer buttons. Using these times we constructed two response variables of interest: *First Action Time* and *Last Action Time*. Respectively, these are the first and last times the subject interacts to answer the question (whether by click or keystroke), excluding the final click-/keystroke to submit their answer.

We presented the survey to workers on Amazon's Mechanical Turk[2]. To quality for the main experiment, subjects had to pass a 3 question Java code comprehension task. Answering all 3 questions correctly allowed them to choose to continue to the main task and the instructions for it. After the subjects completed the main task, there was an optional demographics survey and a couple open ended feedback questions. At the end, they were presented their overall score on the main task, and we exclude from data analysis any response that received a score of less than 20 out of 32.

Participants

We restricted our participants to workers on Mechanical Turk who had 1000 or more hits, a 99% or greater acceptance rate, and were from the US or Canada. We also used Unique Turker[3] to avoid duplicate attempts. In total, we had 343 attempts on the qualification task, and 116 full completions of the main task with 111 scoring 20 or higher.

The subjects reported an age with mean 32.2 and s.d. 8.8 years, Java experience with mean 10.5 and s.d. 5.3 years, and took 34.2 minutes on average with s.d. 14.3. About 67% programmed at least a few times a week and most of the rest at least at few times a month. Almost all participants had at least at some college education, with over 50% having a Baccalaureate degree. Most use Mechanical Turk as an extra source of income. We paid $1 to everyone who took the qualification (pass or fail), and an additional $4 to everyone who completed the main task, regardless of score.

Results

Statistical Analysis

We have 3 primary response variables of interest, 1) a binary variable for whether the respondent answered the question correctly, 2) their *First Action Time* in seconds, and 3) their *Last Action Time* in seconds. Our primary explanatory variable for surprisal is a binary value which is 0 if this variant was the less surprising version or 1 if it is the more surprising version. We analyze our data using mixed-effects regression models. Our full models contain fixed effects for which of the 4 transformation types a question was, and whether the question was a text box or true/false question. We also considered interaction effects of each of these with the surprisal value. Our random effects are the maximal structure justified by the design(Barr, Levy, Scheepers, & Tily, 2013); for items, we have a random intercept and a slope for surprisal; for subjects, a random intercept and slopes for surprisal, transformation type, question type, and their interactions. We use deviation coding for all categorical variables; each coefficient is in comparison to the grand mean. Therefore, for example, the regression formula for our full model using the binary measure for *Last Action Time* is as follows: *Last Action Time* $\sim$ *Surprisal*(TransType + QuestionType) + (1 + Surprisal*(TransType + QuestionType)|ResponseID) + (1 + Surprisal|Question)*. The correctness models were logistic regressions, and the timing models were lognormal, as we observed that fit the distribution of the response well. We fit these models using the brms package for bayesian regression, using default priors (Bürkner, 2017). We also tested each of these full models against simpler ones to check robustness, using WAIC scores to compare them (Watanabe, 2010). When the simpler models have qualitatively similar results to the full models, but much better WAIC scores, we present the simpler models.

As these experimental methods have not typically been applied to code, we additionally want to explore best practices for these types of experiments, so we also considered our data in a few other ways. We considered our models with and without timing outliers (cases where subjects answered more

[1] https://developer.mozilla.org/en-US/docs/Web/API/Performance

[2] https://www.mturk.com/

[3] https://uniqueturker.myleott.com/

than 3 standard deviations away from the mean of first and last action time), but observed that they had minimal impact, so we present the models with outliers. We also modeled the text answer and true/false questions separately, after observing different behavior from these questions in our models and plots. Supplementary materials and R notebooks showing models and plots not included in results can be found in the anonymous archive: https://doi.org/10.5281/zenodo.3626129

Timing and Surprisal

Table 2: Fixed effects for bayesian mixed effects lognormal regression comparing if a variant was more or less surprising against the *Last Action Time*. WAIC scores suggested the model without interactions was best.

	Estimate	Error	1-95% CI	u-95% CI
Intercept	3.08	0.06	2.96	3.20
Surprisal	0.07	0.02	0.02	0.12
AddParen	0.09	0.09	-0.08	0.26
Arithmetic	-0.20	0.09	-0.37	-0.03
Relational	0.02	0.09	-0.17	0.19
Text	0.15	0.05	0.04	0.26

First, let's consider the question of whether more surprising code takes longer for humans to comprehend. We will focus primarily on *Last Action Time*, as the effects were larger and more significant than *First Action Time*. Figure 2a shows the median time difference for *Last Action Time* plotted against the difference in surprisal. Plotting lines from very simple regressions trend upwards as predicted; participants tended to answer questions about the relatively more surprising version of the code more slowly.

Now, to see if these effects are significant, we present the results of our mixed effects models which account for the variance of the questions and subjects. We will discuss the Last Action model in detail in Table 2, and then briefly mention the First Action model. Surprisal has a significant effect, and as the regression is lognormal, we can interpret the average variant with lower surprisal taking 21.8 seconds and with higher surprisal taking 23.3 seconds. Looking at the other coefficients, we see significant differences on arithmetic questions and on text questions, with arithmetic being significantly faster and text questions significantly slower. In comparison to this model, the estimated surprisal coefficient for the *First Action Time* model with best WAIC score is 0.06 with 95%-CI (0.00, 0.11). The effect is suggestive, but not large enough to conclude significance.

When we divide our data to look at the text and true/false questions separately in models, we see that the the effect is less significant for the true/false models, but much more so in the text models. The text questions exhibit larger effects; in fact, for the text questions only both first and last action time show significant effects of surprisal: 0.12 with CI (0.01, 0.23) for the first action model and 0.14 with CI (0.07, 0.21) for the last action model. However, when modeling the true/-false questions only, the effects are no longer significant - both contain 0 within their credible intervals. In Figure 3a, we can see these trends in a summarized version of the data with simple regression models.

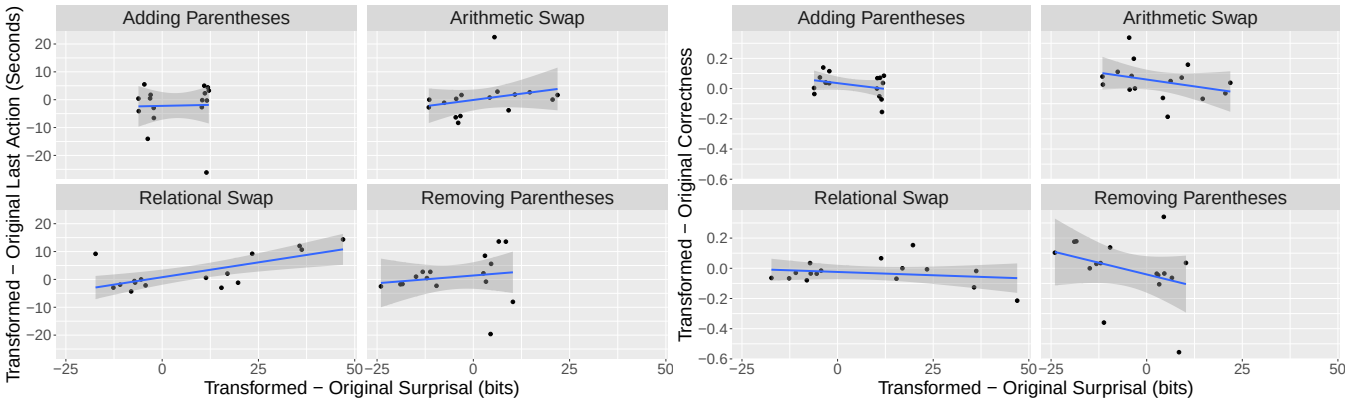
Case Studies Some transformations had drastic changes in time; using the difference in median last action time, we present the most extreme examples for and against our hypothesis that higher surprisal leads to longer mental computation. For the most extreme example that agreed with our theory, we saw an increase of *22.5* seconds when changing `time -= hours * 60 * 60;` to `time -= 60 * hours * 60;`. The language model preferred the original in this case. The most extreme change in the direction opposite to what is predicted by our theory also happened when the language model gave lower surprisal on the original, but the transformation proved much easier to comprehend. By changing the original code `for(int i = 0; i < _maxfev && step >= minStep; ++i, step *= _stepDec)` by adding parentheses around `step >= minStep`, subjects were able to correctly determine how many times the loop executed a median of *26.1* seconds faster. Small line level transformations can *substantially* change how quickly humans correctly comprehend code.

Accuracy and Surprisal

For our mixed effects model of correctness, the best model judging by WAIC scores was a model with no interactions that excluded the transformation type as a categorical variable. The binary surprisal coefficient was -0.21, suggesting a negative trend between surprisal and correctness. However, the 95% credible interval on this coefficient was (-0.56, 0.15). As this interval is wide and includes 0, we cannot conclusively say that the effect is significant, though it is at least in the expected direction. Graphically, we express this trend in Figure 2b, comparing the accuracy against the surprisal difference, summarized for each question.

As in the timing models, there is a significant difference between the text and true/false questions, so we divide the data and model them separately. We see that the the effect is even less significant for the true/false models, but more so in the text models. The coefficient for surprisal for the text only model is -0.37, but the 95% credible interval still contains 0 (-0.87, 0.11). Figure 3b shows a summarized plot of these trends. Therefore, we can at best say the effects are suggestive but not conclusive for correctness; further study is needed to link surprisal and comprehension accuracy.

Case Studies As with timing, we present the most extreme cases for and against our hypothesis that higher surprisal means fewer correct responses for the questions. For an example of the largest change in the expected direction, we swapped the operands in the expression `res[numstart + i] += scale * numVals[i];` to change it to `res[numstart + i] += numVals[i] * scale;`. The transformed code had lower surprisal, and it im-



(a) Median Last Action Time vs Surprisal Difference

(b) Correctness vs Surprisal Difference

Figure 2: Per question pair plots of a) last action time and b) question accuracy differences against the difference in surprisal between the transformed and original code broken down by each of the 4 transformations. For the x axis on each plot, further left indicates when original code is more surprising, and further right means the transformed code is more surprising. For a), the y axis plots the difference in median time between the two; smaller values (below 0) indicate that questions about the transformed code were answered more quickly. For b) the y axis plots the difference in fraction of correct answers. Higher values mean more people correctly answered questions about the transformed code.

proved the percent of correct answers from 55.5% to 89.2%. In this case, we theorize that grouping the array accesses together might have made the mental computation easier. The most extreme change in the unexpected direction was the transformation from `return (2.0 / sampleSize) * (prediction - lb);` to `return 2.0 / sampleSize * (prediction - lb);`. The transformation had lower surprisal, but 93.1% of our subjects correctly computed the original code and only 57.1% did so for the transformed version. Perhaps the lack of parentheses made the order of operations between the divide and multiply operations unclear.

Discussion

Does higher language model surprisal predict increased difficulty in processing code? Our experiment provides suggestive but not definitive evidence for this effect. The clearest effects appeared in the models measuring the total time to answer the comprehension questions; for models measuring the first time the subjects interacted with the questions and whether the question was answered correctly, the effects trended in the predicted direction but were not significant. Exploratory analysis beyond our main models showed these effects were more pronounced when we only considered text box questions and excluded true/false questions.

One may question whether such small changes to single lines of code could really affect how easy they are for humans to understand. However, our case studies demonstrated this is not true by counterexample; we found cases where even a small change could drastically alter how quickly and accurately participants could answer questions. A single changed parentheses or reordered statement could lead to a misunderstanding about some code's meaning.

Our experiment shows comprehension tasks are a viable method of studying how people process code, and provides some recommendations for future studies of this type. First, focus on comprehension questions that require text entry answers. True/false questions can be more easily guessed, and might be too simple to see the desired effects. Likewise, more difficult questions may be able to create more significant effects. Finally, we hope that as these effects are better understood, it may be possible to use surprisal as a method to guide automated tools to modify code to be more easily understandable by humans without altering its computational meaning.

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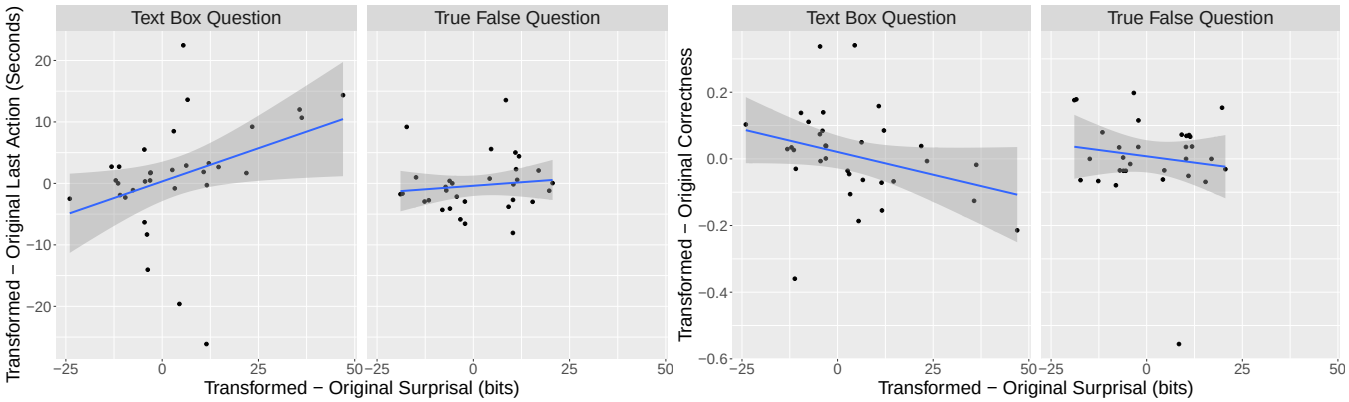

(a) Median Last Action Time vs Surprisal Difference

(b) Correctness vs Surprisal Difference

Figure 3: Per question pair plots of a) last action time and b) question accuracy differences against the difference in surprisal between the transformed and original code broken down by each of 2 types of questions. For the x axis on each plot, further left indicates when original code is more surprising, and further right means the transformed code is more surprising. For a), the y axis plots the difference in median time between the two; smaller values (below 0) indicate that questions about the transformed code were answered more quickly. For b) the y axis plots the difference in fraction of correct answers. Higher values mean more people correctly answered questions about the transformed code.

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